AD-A038 592

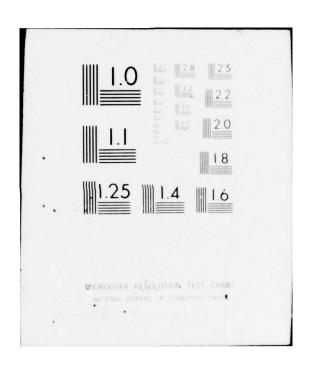
DAVID W TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CE-ETC F/G 17/7 PROBABILITIES OF DETECTION AND IDENTIFICATION OF NAVIGATION BUO-ETC(U) FEB 77 H FEINGOLD, D RITTER, J TOZZI

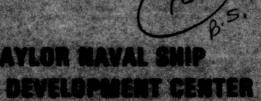
UNCLASSIFIED

TOTNSRDC-77-0031

NL

END
DATE
FILMED
5-77







AD A U 38592

PROBABILITIES OF DETECTION AND IDENTIFICATION
OF NAVIGATION BUOY LIGHT SIGNALS

Harry Feingeld (DTNSRDC)

Dougles River (DTNSRDC)

Lt. John Terri (U.S. Court Guird)



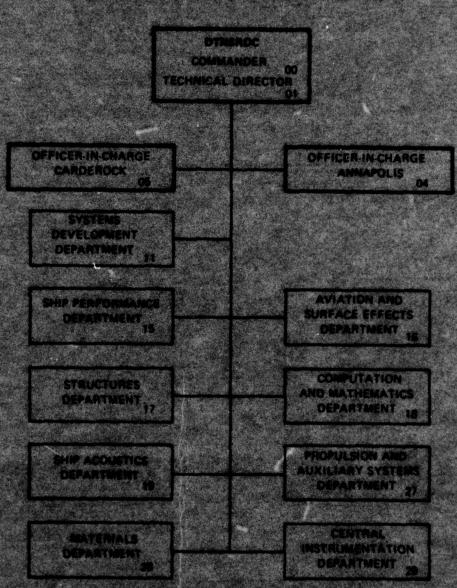
AUGROVED SON CHOICE BELEASE: DISTRIBUTION LAKE HATED

THE COPY

COMPUTATION, MATHEMATICS, AND LOGISTICS DEPARTMENT RESEARCH AND DEVELOPMENT REPORT

Emer 77,0001

MAJOR OTHERDS GREANIZATIONAL COMPONENTS



UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

REPORT DOCUMENTATION		READ INSTRUCTIONS BEFORE COMPLETING FOR
I. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
DTNSRDC -77-0031		
4. TITLE (and Subtitle)	The state of the s	5. TYPE OF REPORT & PERIOD COV
PROBABILITIES OF DETECTION AND J		Final rest.
NAVIGATION BUOY LIGHT S	SIGNALS.	6. PERFORMING ORG. REPORT NUMB
		8. CONTRACT OR GRANT NUMBER(s)
Harry/Feingold, (DTNSRDC)		TO SOM THE SOM STAN THOMBEN
Douglas/Ritter (DTNSRDC)		
John/Tozzi/(U.S. Coast Guard	1)	
9. PERFORMING ORGANIZATION NAME AND ADDR		10. PROGRAM ELEMENT, PROJECT, 1
David W. Taylor Naval Ship Resea	irch v	Task Area: Gov't.
and Development Center		Work Unit: 1-1576-130
Bethesda, Maryland 20084		12. REPORT DATE
11. CONTROLLING OFFICE NAME AND ADDRESS	. (1)	February 1977
Navigation System Technology Bra	anch (1)	13. NUMBER OF PAGES
U.S. Coast Guard		24 (12) 266.)
14. MONITORING AGENCY NAME & ADDRESS(II dill	ferent from Controlling Office)	15. SECURITY CLASS. (elfate report)
		UNCLASSIFIED
		15. DECLASSIFICATION DOWNGRAD
16. DISTRIBUTION STATEMENT (of this Report) APPROVED FOR PUBLI	C RELEASE: DISTRI	BUTION UNLIMITED
		D D C
APPROVED FOR PUBLI		D D C
APPROVED FOR PUBLI		D D C
APPROVED FOR PUBLI		D D C
APPROVED FOR PUBLI		D D C
APPROVED FOR PUBLI 17. DISTRIBUTION STATEMENT (of the abstract entre 18. SUPPLEMENTARY NOTES	ered in Block 20, if different from	m Report) APR 26 197
APPROVED FOR PUBLI 17. DISTRIBUTION STATEMENT (of the abstract ento 18. SUPPLEMENTARY NOTES 19. KEÝ WORDS (Continue on reverse elde if necesses)	ered in Block 20, if different from	m Report) APR 26 197
APPROVED FOR PUBLI 17. DISTRIBUTION STATEMENT (of the abstract ento 18. SUPPLEMENTARY NOTES 19. KEÝ WORDS (Continue on reverse elde if necesses Buoys Di	ered in Block 20, if different from ry and identify by block number)	m Report) APR 26 197 Apr 26 197 utocorrelation
APPROVED FOR PUBLI 17. DISTRIBUTION STATEMENT (of the abstract ento 18. SUPPLEMENTARY NOTES 19. KEÝ WORDS (Continue on reverse elde II necessai Buoys Probability of Detection Ro	ered in Block 20, if different from ry and identify by block number) ivergence Angle Ai oll and Pitch Di	m Report) APR 26 197
APPROVED FOR PUBLICATION STATEMENT (of the abstract ento) 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse elde II necessal Buoys Probability of Detection Roman Probability of Identification To	ored in Block 20, if different from the second in Block 20, if different from the second in Block number) ivergence Angle Alboral Tilt Angle	m Report) APR 26 197 Apr 26 197 utocorrelation
APPROVED FOR PUBLICATION STATEMENT (of the abstract entering to the abs	ry and identify by block number) ivergence Angle At old and Pitch Di otal Tilt Angle tep Response Data	utocorrelation irection of Tilt
APPROVED FOR PUBLICATION STATEMENT (of the abstract entering to the abs	ry and identify by block number) ivergence Angle Atoliand Pitch Dibatal Tilt Angle tep Response Data y and identify by block number)	utocorrelation irection of Tilt
APPROVED FOR PUBLIC 17. DISTRIBUTION STATEMENT (of the abstract enter 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse elde II necessar Buoys Di Probability of Detection Ro Probability of Identification To Light Characteristic Flashes St 20. ABSTRACT (Continue on reverse elde II necessar The position in azimuth of	ry and identify by block number) ivergence Angle Atoliand Pitch Distal Tilt Angle tep Response Data y and identify by block number) an observer relative	utocorrelation irection of Tilt
APPROVED FOR PUBLIC 17. DISTRIBUTION STATEMENT (of the abstract ento) 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse elde II necesses Buoys Di Probability of Detection Ro Probability of Identification To Light Characteristic Flashes St 20. ABSTRACT (Continue on reverse elde II necesses The position in azimuth of motion (tilt) of a buoy most sig	ry and identify by block number) ivergence Angle At old Tilt Angle tep Response Data y and identify by block number) an observer relating	utocorrelation irection of Tilt ve to the plane of angula the ability of an observ
APPROVED FOR PUBLIC 17. DISTRIBUTION STATEMENT (of the abstract enter 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse elde II necessar Buoys Di Probability of Detection Ro Probability of Identification To Light Characteristic Flashes St 20. ABSTRACT (Continue on reverse elde II necessar The position in azimuth of	ry and identify by block number) ivergence Angle At old Tilt Angle tep Response Data y and identify by block number) an observer relating gnificantly affects entify specific buo	utocorrelation irection of Tilt ve to the plane of angula the ability of an observy characteristic flashes.
APPROVED FOR PUBLIC 17. DISTRIBUTION STATEMENT (of the abstract enter 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse elde II necessar Buoys Probability of Detection Probability of Identification To Light Characteristic Flashes St 20. ARSTRACT (Continue on reverse elde II necessar The position in azimuth of motion (tilt) of a buoy most sig to initially detect and then ide Under idealized assumptions it is records of buoy angular motion of	ry and identify by block number) ivergence Angle At old Tilt Angle tep Response Data y and identify by block number) an observer relative gnificantly affects entify specific buoy is possible to deve	utocorrelation irection of Tilt ve to the plane of angula the ability of an observy characteristic flashes. lop from appropriate past
APPROVED FOR PUBLIC 17. DISTRIBUTION STATEMENT (of the abstract enter 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse elde II necessar Buoys Probability of Detection Probability of Identification To Light Characteristic Flashes St 20. ARSTRACT (Continue on reverse elde II necessar The position in azimuth of motion (tilt) of a buoy most sig to initially detect and then ide Under idealized assumptions it is records of buoy angular motion of	ry and identify by block number) ivergence Angle At old and Pitch Di otal Tilt Angle tep Response Data y and identify by block number) an observer relating gnificantly affects entify specific buoy is possible to deve	utocorrelation irection of Tilt ve to the plane of angula the ability of an observy characteristic flashes. lop from appropriate past

DD 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE S/N 0102-014-6601

UNCLASSIFIED

387682 Property Security CLASSIFICATION OF THIS PAGE (Phon Data Entered)

LECHHITY CLASSIFICATION OF THIS PAGE(When Date Entered)

(Block 20 continued)

While these results were developed under highly idealized assumptions, it is recommended that further studies be conducted with more realistic assumptions that take into account: (1) autocorrelation of buoy tilt angle in relation to buoy identification time, (2) central tendency of buoy roll and pitch, (3) the lobeshaped light divergence pattern as contrasted to the wedge-shaped divergence pattern, and (4) the physiological and psychological factors that affect detection and identification.

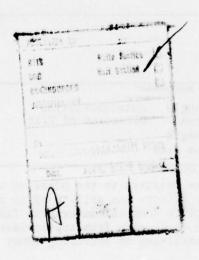


TABLE OF CONTENTS

			Page
ABS	TR	ACT	1
1.		INTRODUCTION	1
II.		THE PROBLEM	4
111		PROBLEM SOLUTIONS	7
	Α.		7
	В.		8
	c.	PROBABILITY OF IDENTIFICATION	13
IV.		DISCUSSION AND RECOMMENDATIONS	18
		LIST OF FIGURES	
la	-	A Counterweight-Tube Buoy	2
1b	-	Flat-Bottom Buoy	2
2 a	-	Standard A N Buoy Signal Lantern	5
2b	-	Vertical Distribution of Luminous Intensity (Not Drawn to Scale)	5
3a	-	Idealization of Vertical Intensity Distribution	6
3b	-	Effect of Angular Motion of Buoy Upon Idealized Light Pattern	6
4a	-	Tilted Buoy Signal as Seen From Above. γ is the Direction of Tilt as Defined by Equation (9)	10
4b		Tilted Buoy Signal Lamp as seen in the Plane of Angular Motion. δ is the Angle of Tilt as Defined by Equation (5)	10
5 a	-	Area Covered by Buoy Light (Top View). W is Length of Cord of Sector Covered By Light	12
5b	-	Area Covered by Light (Side View)	12
6 a	-	Hypothetical Time Records of Roll Angle and Pitch Angle	15
6b	-	Time Records of (a) Above Using δ and γ Obtained from Equations (5) and (9), Respectively	15
6c	-	Time Record of γ from (b) Above Time Record of β Obtained from γ in (b) Using Equation (6)	15

							Page
6d	-	Angular Boundaries in the Check Light can be seen					16
6e	-	Hypothetical Characteristics Flash Sequence					16
6f	-	Illustration of Practical Calibration of η for Sequence of Characteristic Flashes					16
7a	-	Situation During First Flash (t ₀). Shaded Area is Covered by Light				7	17
7b	-	Situation During 2nd Flash (t ₁). Shaded Area is Covered by Light					17
7c	-	Combination of (a) and (b) Above Showing Area in Which Both Flashes can be seen					17

ABSTRACT

The position in azimuth of an observer relative to the plane of angular motion (tilt) of a buoy most significantly affects the ability of an observer to initially detect and then identify specific buoy characteristic flashes. Under idealized assumptions it is possible to develop from appropriate past records of buoy angular motion data, probabilities of detecting (POD) and identifying (POI) buoys.

While these results were developed under highly idealized assumptions, it is recommended that further studies be conducted with more realistic assumptions that take into account: (1) autocorrelation of buoy tilt angle in relation to buoy identification time, (2) central tendency of buoy roll and pitch, (3) the lobe-shaped light divergence pattern as contrasted to the wedge-shaped divergence pattern, and (4) the physiological and psychological factors that affect detection and identification.

I. INTRODUCTION

The U.S. Coast Guard has in operation approximately 14,000 lighted aids to navigation which serve water-borne traffic on the navigable waters of the United States. Of these aids, about 4,000 are lighted buoys. There are six major hull designs for lighted navigation buoys in service today. These designs are of two major types: counterweight-tube buoys and flat-bottom buoys (Figure 1). Each design type reflects clearly the intended use of the buoy. Buoys with counterweight tubes are intended for use in areas of relatively deep water; flat-bottom buoys are more suitable in areas of shallow water. Smaller buoys are used in more sheltered areas and larger ones serve in more exposed areas, i.e., larger buoys are expected to withstand rougher sea conditions.

The payload of a lighted navigation buoy consists primarily of its light signal and a supporting energy source. Buoy maintenance costs can

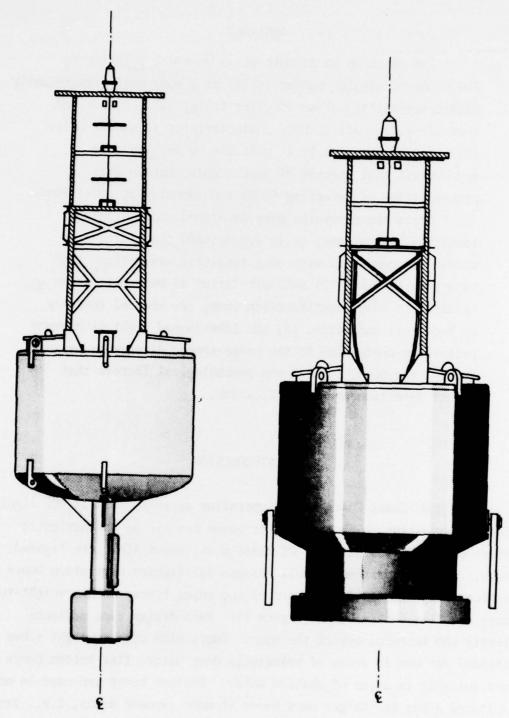


Figure la - A Counterweight-Tube Buoy Figure lb - Flat-Bottom Buoy

Figure 1

be reduced significantly by reducing the required servicing frequency of the buoy. Servicing frequency is determined by (1) mooring life, (2) hull coating life, and (3) energy source life. For a long time, the major limiting factor has been the energy source life. The demands on the energy source are determined by the intensity of the light required to provide the desired signal range. This range can be increased substantially by making the light directional, i.e., by focusing it in a desired direction. If a light is made directional, less power is required for a given range. This permits a reduction in the energy requirement of the light and, for the same size energy source, increases the source life. For this reason, all Coast Guard navigation buoy lights are directional.

The degree of directionality is prompted only by cost considerations. The combined impact of buoy motion and light directionality (lens divergence angle) on the effectiveness of the light signal is considered only intuitively. However, the increased navigational accuracy requirements of large, fast tankers and cargo ships emphasize the need for a more quantitative assessment of the influence of these factors on signal effectiveness.

The six major lighted buoy hull designs in the Coast Guard inventory have not been altered significantly in the last forty years. The small design modifications which have been made were prompted primarily by cost considerations. Recent improvements in plastics construction have encouraged the Coast Guard to consider seriously the possible savings in procurement and maintenance costs in using plastic material for buoy construction. The use of plastics, whose mechanical properties are obviously much different than those of steel, will require the development and use of new, different buoy hull designs to accommodate different construction techniques. Consequently, there is a need for a quantitative analytical procedure for comparing different designs.

The purpose of this study is to consider the problem of developing quantitative methods for determining the probability of detection (POD) and the probability of identification (POI) of a navigation buoy light signal, given a set of reasonable assumptions. Some logical extensions

of the theory which permit a more general development are discussed in Section III (Recommendations and Discussion).

II. THE PROBLEM

A typical buoy signal lamp system used operationally as an aid-to-navigation is shown in Figure 2a. The vertical distribution of luminous intensity due to the lens is shown qualitatively in Figure 2b. In practice, the "lobe" is generated theoretically by plotting intensity versus angle on polar-log graph paper. The U.S. Coast Guard defines the divergence angle of the lens as the angle between the 50%-of-maximum-intensity points.

Since the lamp is rigidly attached to the buoy, the motions of the buoy in a seaway cause definite vertical, lateral, and angular motions of the lamp. Of these the angular motions most significantly affect the ability of an observer to detect and identify the signal. The idealization of the vertical intensity distribution shown in Figure 3a is useful in visualizing the true effect of the angular motion on the signal. The figure depicts the light beam emitted from the lamp as a three-dimensional solid which is generated by revolving a circular sector, defined by the divergence angle and the maximum range of the light (determined by maximum intensity), about the centerline axis of the buoy. Consider an angular rotation of the lamp of δ , where δ is greater than the divergence angle α (Figure 3b). Obviously, depending on his position in azimuth with respect to the plane of tilt, an observer will or will not see the signal. The vertical motion of the buoy contributes little to the detectability of the light, since the vast majority of buoy signals are power limited in range, not horizon limited; and lateral motions are not large enough to affect detectability.

All buoy signal lamps have a definite characteristic flash which further complicates the problem. The human eye does not react instantaneously to light impinging upon it. The process of seeing a light of a given intensity involves a certain integration time. The problem



Figure 2a - Standard A N Buoy Signal Lantern

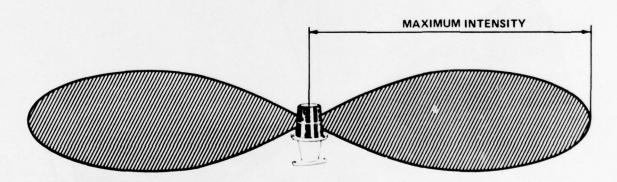


Figure 2b - Vertical Distribution of Luminous Intensity (Not Drawn to Scale)

Figure 2

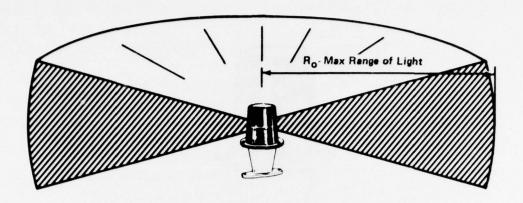


Figure 3a - Idealization of Vertical Intensity Distribution

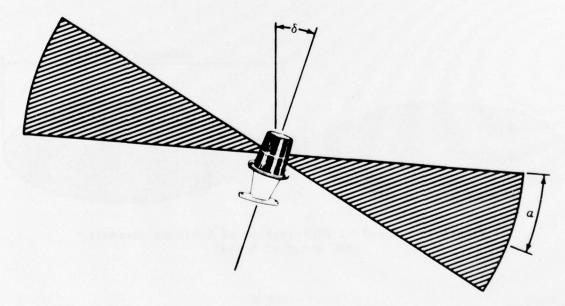


Figure 3b - Effect of Angular Motion of Buoy Upon Idealized Light Pattern

Figure 3

can be considered as the attenuation of the light intensity as a function of the time of exposure of the eye to the light. This phenomenon modifies the effective range of the light and thus affects the measures of POD and POI.

Now consider the effect of buoy motions on the idealized light of Figure 3a. (See Figure 3b.) The movement of the light in and out of the field of vision of an observer creates an additional flash effect (called apparent flash). Consequently, when the characteristic flash is combined with the motion of the buoy, it may be impossible for certain observers to detect the signal at all. Further, buoy motion can reduce the probability of identification by causing an observer to miss one or more of the flashes in a sequence so that, although he may detect the signal, he may not be able to identify it.

As ment aned previously, the position in azimuth of an observer relative to the plane of angular motion (tilt) is critical to his chances of detecting or identifying the signal. Further, the observer's height of eye will affect the relative visibility of the signal. Geographical and weather conditions as well as psychophysical factors will also affect the probabilities of detection and identification of the signal.

These factors make the problem extremely complicated. In its present form, it does not lend itself to any reasonable closed-form solution. Consequently, certain simplifying assumptions were made to make the problem tractable. These assumptions are discussed in detail with each theoretical development in the next section.

III. PROBLEM SOLUTIONS

A. DEFINITIONS

The Probability of Detection (POD) is defined as the probability that an observer will see the buoy light on any given instantaneous observation provided that he is within the range of the lamp and that he is looking in the general direction of the buoy. It is a simple, idealized measure of signal effectiveness.

The Probability of Identification (POI) is defined as the probability that an observer will see enough of the signal characteristic to permit him to identify the buoy from which the signal is emitted. Obviously, the POI restricts the possible simplification of the problem more than the POD does.

This section discusses the theoretical development of both POD and POI. All significant idealizing assumptions are stated and the developments are given in sufficient detail to enable the complete theoretical development and implementation of both methods for determining signal effectiveness.

B. PROBABILITY OF DETECTION

1. Assumptions

The definition of POD permits the following idealizing assumptions with respect to the buoy:

- The vertical variation of luminous intensity is as shown in Figure 3a.
- The light shines continuously, i.e., there is no actual flash.
 - Only angular positions of the light affect its visibility.
- The maximum range of the signal is power limited, not horizon limited, and is not affected by apparent flash.
- The light is never obstructed by the sea surface or the buoy structure.

Most of the significant assumptions relate to the observer. The observer is assumed to be on a platform which moves laterally and vertically in exact correspondence to the lateral and vertical motions of the buoy. No other motions of the observer are permitted. The eyes of the observer are assumed to be in the focal plane of the lamp. It is assumed that the available buoy motions data can be processed to yield the joint probability density function of roll angle (θ) and pitch angle (ϕ) . (For axisymmetrical buoys the roll and pitch axes are orthogonal but no absolute direction is specified for either.) However, it is assumed further that the tilt direction cannot be related unambiguously to a specific geographical direction due to the yawing of

the buoy. Therefore, the position of the observer in azimuth relative to the plane of the buoy's angular motion is assumed to be equally probable at any position around the buoy. This is equivalent to fixing the position of the observer and assuming that the direction of tilt is equi-probable at any position around the buoy. The observer's position is, of course, assumed to be within the maximum signal range of the buoy.

2. Theory Development

The POD theory is developed under the assumptions stated above. In equation form, POD is defined as:

POD =
$$\sum_{i j} P(\text{seeing the signal} | \theta_i, \phi_j) \cdot P(\theta_i, \phi_j)$$
 (1)

where θ is the roll angle

φ is the pitch angle

P(seeing the signal $|\theta_i, \phi_j|$ is the probability of seeing the signal given θ_i, ϕ_j

$$P(\theta_i, \phi_j)$$
 is the joint probability that $\theta = \theta_i \pm \frac{\Delta \theta}{2}$ and $\phi = \phi_j \pm \frac{\Delta \theta}{2}$

As stated above, it is assumed that the joint probability density function of θ and φ is obtainable from the available buoy motion data. (The exact form of the joint density function does not affect significantly the ease of solution since Monte Carlo sampling will probably be required in any event.) This assumption implies ergodicity since, realistically, only one sample vector function will be available from the data record of pitch and roll.

The probability of seeing the signal, given that a certain instant the buoy is in a specific pitch and roll position, i.e., $\text{Pr}(\text{seeing the light}|\theta_i,\phi_j) \text{, can be derived directly from geometric considerations. Consider the representation of the buoy as a unit vector in three-dimensional space (see Figure 4). Note that the projection of the vector onto the x-y plane has magnitude <math>\sin \delta$ and direction γ . Therefore the x-component is $(\sin \delta)(\sin \gamma)$, the y-component is $(\sin \delta)(\cos \gamma)$, and the z-component is $(\cos \delta)$. The vector is

$$\hat{B} = (\sin \delta \sin \gamma, \sin \delta \cos \gamma, \cos \delta)$$
 (2)

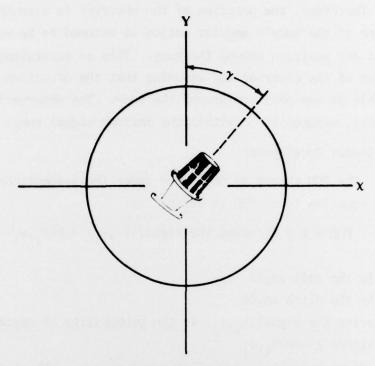


Figure 4a - Tilted Buoy Signal Lamp as Seen from Above γ is the Direction of Tilt as Defined by Equation 9



Figure 4b - Tilted Buoy Signal Lamp as seen in the Plane of Angular Motion δ is the Angle of Tilt as Defined by Equation 5

Figure 4

or

$$\hat{B} = (x^2 - component, y - component, z - component)$$

We define angular motion about the y-axis as pitch and that about the x-axis as roll. So

$$\tan \phi = x/z = \sin \delta \sin \gamma / \cos \delta = \tan \delta \sin \gamma$$
 (3a)

$$\tan \theta = y/z = \sin \delta \cos \gamma / \cos \delta = \tan \delta \cos \gamma$$
 (3b)

Therefore

$$\tan^{2}\phi + \tan^{2}\theta = \tan^{2}\delta \sin^{2}\gamma + \tan^{2}\delta \cos^{2}\gamma$$

$$= \tan^{2}\delta(\sin^{2}\gamma + \cos^{2}\gamma) = \tan^{2}\delta$$
(4)

and consequently

$$\delta = \arctan\sqrt{\tan^2\theta + \tan^2\theta} \tag{5}$$

In order to find the area covered by the light rays, consider the illustration of Figure 5. The reference frame has been reoriented so that the buoy is shown tilted in the plane of the paper, making the method more understandable and the calculations simpler. The buoy light emits rays which hit an assumed sphere of radius R_{0} where R_{0} is the maximum range of the light. Note that if the angle δ is less than α , then the light is visible from any position in the horizontal plane. In this situation

P(seeing the light
$$|\delta \le \alpha$$
) = 1

Now, if $\delta > \alpha$, we refer to Figure 5a and note that W = $2R_O$ sin β . But, from Figure 5b, W sin δ = $2R_O$ sin α ; and, consequently,

$$(2R_0 \sin \beta) \sin \delta = 2R_0 \sin \alpha$$

Therefore,

$$\sin \delta = \frac{\sin \alpha}{\sin \beta}$$
 or $\sin \beta = \frac{\sin \alpha}{\sin \delta}$ (6)

In fact, under our assumptions, P(seeing the light $|\theta_i,\phi_j\rangle$ is merely $\frac{4\beta}{2\pi}$. So, using Equations (5) and (6), for P(seeing the signal $|\theta_i,\phi_j\rangle$, under the stated assumptions, the final equation is

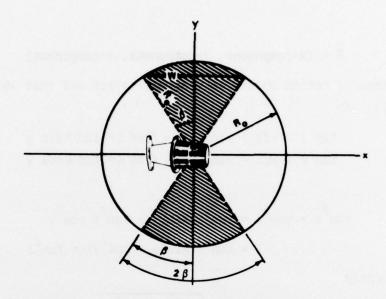


Figure 5a - Area Covered by Buoy Light (Top View)
W is Length of Cord of Sector Covered by Light

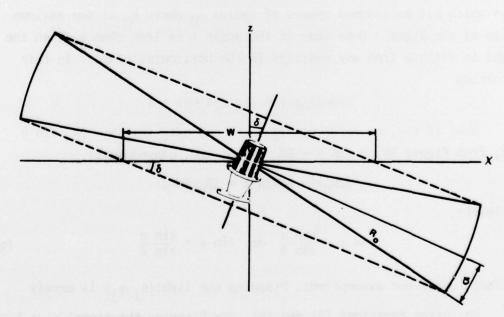


Figure 5b - Area Covered by Light (Side View)

Figure 5

P(seeing the light|
$$\theta_i, \phi_j$$
) = $\frac{2}{\pi} \arcsin \left[\frac{\sin \alpha}{\sin(\tan^{-1} \sqrt{\tan^2 \phi + \tan^2 \theta})} \right]$ (7)

Equation (7) can be used with the expression for $P(\theta_i, \phi_j)$, i.e., the joint probability of the θ_i, ϕ_i , in Equation (1) to yield POD.

C. PROBABILITY OF IDENTIFICATION

1. Assumptions

Development of the theory for POI requires, in addition to the POD assumptions, the assumption that the buoy does not twist significantly about its centerline (Figure 1) during the period of observation. Further, it is assumed that the available data can be processed to yield the true history of total tilt angle (δ) and relative tilt direction (γ) of the buoy and, consequently, the lantern (Figure 4). The concept of identification, unlike that of detection, presupposes both an actual flash and an apparent flash. However, for the purposes of this development, the varying effect of apparent flash upon the effective intensity is assumed constant. Because the range is proportional to the effective intensity, the assumption of constant effective intensity is tantamount to assuming constant range.

2. Theory Development

Since the definition of POI implies an interval of observation, the relative tilt direction must be considered in addition to the total angle of tilt. Therefore, the development begins with the derivation of relative tilt direction as follows:

Dividing Equation (3a) by Equation (3b) yields

$$\tan \phi / \tan \theta = \tan \delta \sin \gamma / \tan \delta \cos \gamma = \tan \gamma$$
 (8)

so that

$$\gamma = \arctan(\tan \phi/\tan \theta)$$
 (9)

Equations (5) and (9) can now be used with the available time records of pitch (ϕ) and roll (θ) to develop the POI theory.

Since the pitch and roll angles are stochastic functions of time, δ , β , and γ are stochastic functions of time, i.e., the stochastic functions $\delta(t)$, $\beta(t)$, and $\gamma(t)$ are directly determinable from the

stochastic functions $\phi(t)$ and $\theta(t)$. Consider, for example, the time records of $\theta(t)$ and $\phi(t)$ shown in Figure 6a. The formulas derived might give the corresponding time records of $\gamma(t)$ and $\delta(t)$ shown in Figure 6b.

The stochastic process $\delta(t)$ and α can be used to generate the stochastic process $\delta(t)$ by Equation (6) (Figure 6c). Consider the situation at $\gamma(t)\pm90^\circ$. By symmetry, either $\gamma(t)\pm90^\circ$ or $\gamma(t)-90^\circ$ could be chosen. For purposes of this discussion, $\gamma(t)-90^\circ$ will be used. The two time records in Figure 6c can be used to yield $\gamma(t)-90^\circ\pm\beta(t)$, which are the boundaries of the areas in which the light can be seen (Figure 6d). The positions of these boundaries are, of course, stochastic processes. At t_0 , observers between $\gamma(t_0)-90^\circ-\beta(t_0)$ and $\gamma(t_0)-90^\circ+\beta(t_0)$ can see the light if it is on at that moment.

Consider three different hypothetical light characteristic flashes as shown in Figure 6e. Superposition of these signals on the boundary function shown in Figure 6d yields the situation shown in Figure 6f. To find the area in which an entire characteristic flash can be seen, find the intersection of the areas within which every flash constituting the characteristic flash can be seen. More specifically, in Figure 6f consider the minimum of $\gamma(t)$ -90°+ $\beta(t)$ over all the flashes within a particular characteristic flash. Similarly, consider the maximum of $\gamma(t)$ -90°- $\beta(t)$ over all the flashes within the same characteristic flash. The maximum and minimum as described above define the borders of the area within which an entire identifying sequence of flashes can be seen. The sequence for one characteristic flash is illustrated in Figure 7. The POI for this particular characteristic flash is given by

$$POI_{CF} = 2\eta/2\pi \tag{10}$$

where

$$\eta = \min_{t_0, t_1} [\gamma(t) \pm 90^{\circ} + \beta(t)] - \max_{t_0, t_1} [\gamma(t) \pm 90^{\circ} - \beta(t)]$$

The POI for a sequence of characteristic flashes might be defined by

POI =
$$\frac{1}{n} \sum_{i=1}^{n} POI_{CF_i}$$

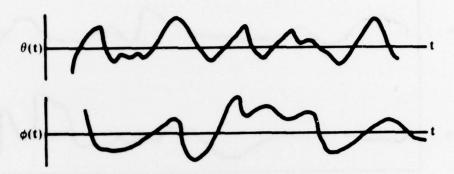


Figure 6a - Hypothetical Time Records of Roll Angle and Pitch Angle

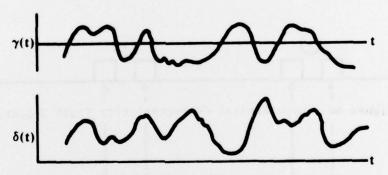


Figure 6b - Time Records of (a) Above using δ and γ Obtained from Equations (5) and (9), Respectively

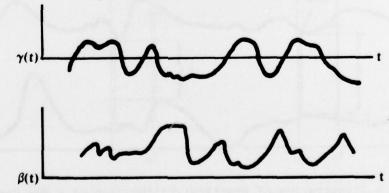


Figure 6c - Time Record of γ from (b) Above. Time Record of β Obtained from δ in (b) using Equation (6)

Figure 6 (Continued)

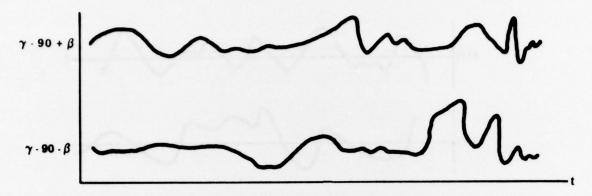


Figure 6d - Angular Boundaries in the Check Light can be seen

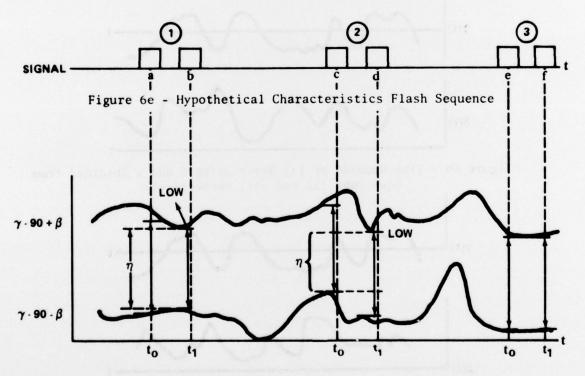


Figure 6f - Illustration of Practical Calibration of η for Sequence of Characteristic Flashes

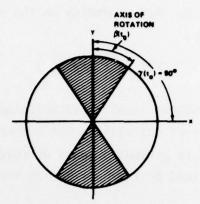


Figure 7a - Situation During First Flash (t_0) Shaded Area is Covered by Light

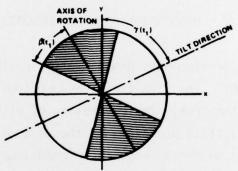


Figure 7b - Situation During Second Flash (t₁)
Shaded Area is Covered by Light

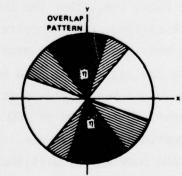


Figure 7c - Combination of (a) and (b) Above Showing Area in Which Both Flashes can be seen

Figure 7

Other definitions could be used depending on the criticality of the buoy to safe navigation, e.g.,

$$POI = \min_{i} [POI_{CF_{i}}]$$

Obviously, the problems involved in defining an overall POI will far exceed the mechanics of calculating the number. The foregoing development is intended to present a means of calculating POI. The overall POI must be defined by the user before the method can be implemented effectively.

IV. DISCUSSION AND RECOMMENDATIONS

The preceding developments demonstrate clearly practical means of obtaining POD and POI. The significant assumptions made for each development are stated in the text. Implementation of either method will require certain subjective decisions on the part of the user. The following discussion addresses some recommendations which are considered pertinent.

Due to the assumed form of the available buoy motions data, the measure of POI developed herein depends strongly on the assumption that there is no yawing of the buoy, i.e., that the buoy does not twist significantly about its centerline. This constraint could be relaxed if some means of determining buoy yaw angle was provided during test deployments. Consequently, buoy yaw angle might be a worthwhile addition to the motions data taken during future test deployments. Furthermore, it may be important to develop specific measures of POD and POI that would be applicable in certain situations. For example, the relative angles between the observer and the buoy may be distributed within very definite limits and with a very well defined central tendency (e.g., buoys in narrow channels). With data that provide the position of the buoy in relation to a fixed reference point, as would be obtained by yaw angle measuring devices, it would be possible to develop such specialized POD and POI measures.

Probabilistic parameters for tilt angle obtained from analyzed step response data might be used in lieu of those obtained from analyzed atsea data, particularly for POD. The validity of such a procedure can be evaluated from the autocorrelation characteristic of the buoy tilt direction. If it can be established that the autocorrelation time is long in relation to the time required for identification of a buoy light signal, buoy angular motion in one dimension (from step response data) may be an adequate form of data input for the determination of POD or POI.

So far, the discussion of the POD and POI measures has been based on the assumption that the divergence of the light is wedge-shaped. Actually, the light divergence shape is best approximated by a lobe-shaped pattern as shown in Figure 2b. Some means of incorporating the actual shape of the light pattern would enhance both methods:

Neither method presented considers the psychological and physiological aspects of the detection and identification problem. It is recommended that further work to incorporate these aspects of the problem into both methods be conducted. Consideration of these aspects might improve the methods significantly.

INITIAL DISTRIBUTION

Copies	
2	USNA 1 Dept Math 1 Library
1	NAVPGSCOL
1	NROTC & NAVADMINU MIT
1	NAVWARCOL
1	NAVSEA 1 SEA 03F/B. Orleans
1	NAVSHIPYD PUGET/LIB
1	NAVSHIPYD CHASN/LIB
1	NAVSHIPYD MARE/LIB
1	NAVSHIPYD NORVA/LIB
1	NAVSHIPYD PEARL/LIB
1	NAVSHIPYD PHILA/LIB
1	NAVSHIPYD PTSMH/LIB
12	DDC
10	HQS COGARD 10 Lt. J. Tozzi (G-DOE-4-TP-54)

CENTER DISTRIBUTION

Copies	Code		Copies	Code	
1	1500		1	1850	
1	1576	W.E. Smith	1	1860	
1	1800		1	1890	
2	1802	Francois M. Frenkiel	30	5214.1	Reports Distribution
		Feodor Theilheimer	1	522.1	Library (C)
1	1805		1	522.2	Library (A)
1	1809.3				
1	1820				
1	1840				
10	184.1				
1	1843				
1	1844				

DTNSRDC ISSUES THREE TYPES OF REPORTS

- (1) DTNSRDC REPORTS, A FORMAL SERIES PUBLISHING INFORMATION OF PERMANENT TECHNICAL VALUE, DESIGNATED BY A SERIAL REPORT NUMBER.
- (2) DEPARTMENTAL REPORTS, A SEMIFORMAL SERIES, RECORDING INFORMATION OF A PRELIMINARY OR TEMPORARY NATURE, OR OF LIMITED INTEREST OR SIGNIFICANCE, CARRYING A DEPARTMENTAL ALPHANUMERIC IDENTIFICATION.
- (3) TECHNICAL MEMORANDA, AN INFORMAL SERIES, USUALLY INTERNAL WORKING PAPERS OR DIRECT REPORTS TO SPONSORS, NUMBERED AS TM SERIES REPORTS; NOT FOR GENERAL DISTRIBUTION.